Department of Mathematics
MAL 250 (Introduction to Probability Theory and Stochastic Processes)
Minor II (I Semester 2013 - 2014)

Time allowed: 1 hour   Max. Marks: 27

1. Let X and Y be random variables with joint *pdf* given by:

$$f(x,y) = \begin{cases} \frac{24x^2}{y^3} & 0 < x < 1; \ y > 2 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find the marginal density function of X.

   (b) Calculate $P\left(X < \frac{1}{2}/Y > 6\right)$.

                                                                (2 + 3 marks)

2. Let A, B and C be independent random variables each with uniform distributed on interval $(0, 1)$. What is the probability that $Ax^2 + Bx + C = 0$ has real roots?   (5 marks)

3. Let X be a random variable which is uniformly distributed over the interval $(0, 1)$. Let Y be chosen from interval $(0, X]$ according to the pdf

$$f(y/x) = \begin{cases} \frac{1}{x} & 0 < y \le x \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find $E(Y^k/X)$ for any fixed positive integer k.

   (b) Find the characteristic function of Y.

                                                                (2 + 3 marks)

4. Using MGFs, find the limit of Binomial distribution with parameters n and p as $n \to \infty$ such that $np = \lambda$ so that $p \to 0$.   (4 marks)

5. Does the random variable X exist for which

$$P[\mu - 2\sigma \le X \le \mu + 2\sigma] = 0.6$$

   Justify your answer.   (2 marks)

6. (a) State central limit theorem.

   (b) Prove the central limit theorem with the assumption of the sequence of iid random variables.

   (c) Let $\{X_i, i = 1, 2, \ldots\}$ be a sequence of iid random variables with mean 10 and standard deviation 4. This sequence of random variables form a population. A sample of size 100 is taken from this population. Find the approximate probability that the sample mean of these 100 observations is less than 9. ($P(Z < -2.5) = 0.0062$, $P(Z < -2.0) = 0.0228$), $P(Z < -1.5) = 0.0668$))

                                                                (2 + 2 + 2 marks)

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